INFLUENCE OF P-DELTA EFFECTS ON SEISMIC DESIGN

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SYNOPSIS

A brief discussion is given of the influence of P-Delta effects on the response of buildings subjected to earthquake ground motion. Time-history studies on the response of shear buildings to two earthquake ground motions are presented. It is shown that the influence of P-Delta effects is of great importance for buildings responding in a highly inelastic manner. However, for buildings responding in an elastic or slightly inelastic manner, the influence of P-Delta effects is relatively small. The stability factor approach for estimating the influence of P-Delta effects is reviewed. It appears that this approach gives reasonable results only for systems responding in an elastic or slightly inelastic manner. The strength and drift characteristics of buildings are briefly described. The results presented suggest that the response of certain types of well-designed buildings will not be significantly influenced by P-Delta effects.

RESUME

Dans cet article on discute de l'influence des effets P-Δ sur la réponse des bâtiments aux séismes. On y présente les résultats d'une étude sur la variation dans le temps de la réponse de bâtiments soumis à deux accélérations du sol. On a constaté que les effets P-Δ ont une grande importance pour les bâtiments qui ont une réponse sismique fortement inélastique. Par contre si la réponse sismique est élastique ou un peu inélastique les effets P-Δ sont relativement faibles. On a révisé la méthode du facteur de stabilité servant à évaluer l'importance des effets P-Δ. En effet il semble que cette méthode ne donne de bons résultats que si la réponse sismique est élastique. On discute brièvement de la résistance et de la flexibilité latérale des bâtiments. Les résultats obtenus laissent supposer que la réponse de certains types de bâtiments bien dimensionnés ne sera pas influencée de façon significative par les effets P-Δ.

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INTRODUCTION

The gravity axial loads acting on columns of buildings during earthquake excitation can have an influence on seismic response when relative story displacements are large. Engineers generally feel that the influence of gravity axial loads, referred to as P-Delta effects in this paper, must be taken into account in the design of certain types of buildings. Unfortunately, practical criteria for judging the significance of P-Delta effects are not available at present.

Past studies on P-Delta effects have been limited to single-degree-of-freedom systems (1) or have considered a small number of multi-story buildings (2). It is the intent of this study to assess the influence of P-Delta effects on the response of a relatively wide range of building designs. In addition, some tentative recommendations are made for assessing the significance of P-Delta effects.

In the first portion of the paper, time-history analysis is used to study the influence of P-Delta effects on the response of shear buildings that are subjected to earthquake base motion. In the second portion, a stability factor procedure that is sometimes suggested for judging the significance of P-Delta effects is reviewed. In the third portion, the P-Delta problem is discussed in the light of practical design considerations.

P-DELTA PROBLEM

In the conventional analysis and design of structures, it is usually assumed that both displacements and strains are small. In practical terms this means that the geometry of the structure remains unchanged during loading. However, it is generally recognized that large displacements and strains will occur when buildings are subjected to major earthquakes. Consequently, the relative story displacements can be large and P-Delta effects will produce additional forces and moments in the structure. These P-Delta effects tend to reduce the structural stiffness. If a building is proportioned so that the strains in the members do not exceed the elastic limit, the reduction in stiffness is small, in general, and the response is not significantly influenced by P-Delta effects (1).

On the other hand, for buildings proportioned so that strains in the members exceed the elastic limit, P-Delta effects can be important. In this case, the structural stiffness is reduced by both P-Delta effects and inelastic response (material nonlinearities). As story displacement increases, the structure eventually reaches a stage where the stiffness becomes zero or even negative. If the duration of the earthquake is long and the building responds in a highly inelastic manner, displacements can be significantly increased as a result of the influence of P-Delta effects.

INFLUENCE OF P-DELTA EFFECTS ON RESPONSE

In this section, the influence of P-Delta effects on the response of one-story, five-story and ten-story shear buildings is quantified. For the shear building idealization, mass is assumed to be lumped at floor levels and structural stiffness is modelled by springs in series connecting adjacent masses. Buildings with flexible columns and very stiff girders, and buildings with certain types of lateral load bracing, are examples of structures for which the shear model is immediately applicable. For this type of model, structural properties can be directly defined in terms of quantities designers commonly work with, such as building weight, W, story shear capacity, V, and yield relative story displacement, Δ_y.

Time-History Analysis

In order to determine the influence of P-Delta effects on response, incremental time-history analysis of the governing equations of motion was used. The analyses were performed using the Newmark Beta Method (3) with $\beta = 1/6$. Within each time step of integration, the stiffness of the building was assumed to remain constant. However, when the deformations at the end of a time step revealed that the structural stiffness must have changed as a result of yielding or hardening, an iterative procedure was used to satisfy the requirements of equilibrium. In the following time step, the new structural stiffness was used. The response was evaluated at the end of each integration step and at each discontinuity in the slope of the ground acceleration history. The size of time step used was 1/20 of the elastic period of vibration for the one-story buildings, 1/20 of the elastic period of the fifth mode of vibration for the five-story buildings and 1/10 of the elastic period of the tenth mode of vibration for the ten-story buildings.

The story shear force-deformation relationships, ignoring P-Delta effects, were assumed to be elastoplastic as shown in Fig. 1. In Fig. 1 $k = V/\Delta_y$ is the elastic or small displacement story stiffness.

The influence of P-Delta effects was physically modelled by adding vertical links (false members) subjected to axial force between each story. With this model, when one floor displaces relative to the floor below, geometric forces are generated by the gravity loads acting on the rigid links. These geometric forces tend to contribute to deflection and must be opposed by the lateral load system. The influence of the geometric forces was incorporated into the analysis by making use of geometric or P-Delta stiffness (4). In Fig. 1 the story shear force-deformation relationship, including P-Delta effects, is obtained by adding the ordinates of the curve entitled "P-Delta effects ignored" to the ordinates of the curve entitled "P-Delta stiffness". The P-Delta stiffness is obtained by dividing the sum of the gravity loads acting at a story, Σw, by the story height, h.

Damping was assumed to be proportional to mass and the coefficient of proportionality between the damping and mass matrices was adjusted so that 5 percent critical viscous damping in the first elastic mode of vibration resulted.

Ground Motions and Buildings Considered

Two ground motions were considered in this study: a pulse motion obtained by taking the first 6.23s of the NS component of the El Centro 1940 earthquake, and the complete accelerogram for the N21E component of the Taft 1952 earthquake. Response spectra for these ground motions, plotted in terms of spectral velocity, are shown in Fig. 2, and the maximum ground motions and durations are tabulated in Table 1.

As mentioned previously, calculations were performed for one-story, five-story and ten-story shear buildings. For the one-story buildings studied, several systems were considered with ratios of building weight to base story shear capacity, W/V, between the limits of 1 and 20. The buildings were proportioned to have yield relative story drifts, $\psi_y = \Delta_y/h$, equal to 0.0025, 0.005 or 0.0075. For the five-and ten-story buildings, systems with the same stiffness, shear capacity, yield relative displacement and height for each story were considered. The total weight of each building was distributed so that the weights of the bottom stories were equal and the weight of the top story was one-half that of one of the bottom stories. Several five- and ten-story systems were considered with ratios of building weight to base story shear capacity, W/V, between the limits of 1 and 20. However, for these buildings only one value of the yield relative story drift, $\psi_y = 0.005$, was considered.

Results

Method of presentation -- In order to study the influence of P-Delta effects on seismic response, two analysis cases were considered for each building. In the first case, each building was subjected to earthquake base motion, and the maximum relative story displacements ignoring the influence of P-Delta effects, Δ_o, were calculated. In the second case, each building was subjected to the same base motion, and the maximum relative story displacements including the influence of P-Delta effects, Δ, were calculated.

The results of the studies are presented in Figs. 3, 4, 5 and 6. In the (a) portion of each figure, the ratios of displacement calculated including the influence to displacement calculated ignoring the influence of P-Delta effects, Δ/Δ_o, are plotted. In the discussion to follow, these plots are used to assess the importance of P-Delta effects. In the (b) portion of each figure, the displacements calculated ignoring the influence of P-Delta effects are plotted in terms of the maximum relative story drift, defined as $\psi = \Delta_o/h$. The advantage of presenting the figures in terms of drift is that the maximum relative story drifts given can be directly compared to the limitations on drift commonly used by design engineers.

Prior to studying the detailed results, it is important to describe the influence on structural properties of varying the ratio of building weight to base story shear capacity, W/V. For a building of given weight, as the ratio W/V increases the flexibility of the structure increases, the strength of the structure decreases and the influence of P-Delta effects becomes more pronounced. Consequently, one would anticipate increasing displacements and greater amounts of inelastic response as the ratio W/V becomes progressively larger.

One-story buildings -- The results of response calculations for one-story buildings subjected to the El Centro pulse and the Taft earthquake are presented in Figs. 3 and 4, respectively. As mentioned above, the figures were constructed for systems with three different values of yield relative story drift. In Fig. 3(a) and 4(a) the increase in displacements as a result of P-Delta effects is shown. It can be seen that the influence of P-Delta effects on the displacement response was small for one-story buildings when the ratio W/V was less than about 5. However, for values larger than 5, the response was significantly influenced by P-Delta effects.

In a past study on P-Delta effects by Jennings and Husid (1), it was demonstrated that the duration of the base motion is an important consideration when inelastic response occurs. In the present study, the El Centro pulse used had a duration of 6.23s whereas the Taft earthquake had a duration of 54.4s. Even though the maximum ground motions were smaller for the Taft earthquake than for the El Centro pulse, Table 1, by comparing Fig. 3(a) to Fig. 4(a) it can be seen that the influence of P-Delta effects was more important for the Taft earthquake. Obviously, duration is an important consideration that was not specifically evaluated in this study.

As expected, when the ratio W/V was varied, the magnitude of displacement response and the amount of inelastic response were significantly influenced. From Figs. 3(b) and 4(b) it can be seen that the maximum relative story drifts became progressively larger as the ratio W/V was increased. By comparing the maximum relative story drifts, ψ, to the yield relative story drifts, ψ_y, the amount of inelastic response that occurred can be assessed. In all cases, for values of the ratio W/V greater than 5, the maximum relative story drifts were larger than 2 times the yield relative story drifts.

In the design of a building to resist earthquake motion, the engineer proportions the structure to satisfy drift requirements. It is important to note that the maximum relative story drifts recorded in Figs. 3(b) and 4(b) were excessive for buildings with large values of the ratio W/V. As discussed in a later portion of the paper, drifts larger than about 0.015 would be considered unacceptable by most design engineers.

Five-story buildings -- In Fig. 5 the results of response calculations for five-story buildings subjected to the Taft earthquake are presented. As mentioned previously, response calculations were performed only for systems with yield relative story drifts of 0.005. Since the displacement response at different levels in a building is of interest, the response values at stories 1, 3 and 5 (story 1 is at the base) are presented as a function of the ratio of building weight to base story shear capacity, W/V.

In Fig. 5(a) the increases in displacements at the three story levels as a result of P-Delta effects are shown. It can be seen that the displacement response was significantly increased only for story 1. In addition, the influence of P-Delta effects was small for values of the ratio W/V less than about 10 or 12.5, and the influence became significant for values of the ratio W/V greater than 15.

It can be seen from Fig. 5(b) that, as the ratio W/V was increased, the magnitude of displacement response and the amount of inelastic response increased. For this earthquake base motion, the largest deformations occurred in story 1. In addition, the amount of inelastic response that occurred was relatively small and for the most part limited to story 1. The maximum relative story drifts for story 1 were larger than 2 times the yield relative story drift only for values of the ratio W/V greater than 10. Furthermore, for the five-story buildings considered the maximum relative story drifts were within reasonable design limits.

Ten-story buildings -- In Fig. 6 the results of response calculations for ten-story buildings subjected to the El Centro pulse are presented. As for the five-story buildings, response calculations were performed only for ten-story systems with yield relative story drifts of 0.005. Since the response of the upper seven stories of the buildings was not significantly influenced by P-Delta effects, only the response values at stories 1, 2 and 3 are presented.

In contrast to the results presented for the five-story buildings, it can be seen from Fig. 6(a) that in some instances for the ten-story buildings the influence of P-Delta effects on response was larger for the upper stories, stories 2 and 3, than for the base story, story 1. In any case, it can be seen that the influence of P-Delta effects was small provided the ratio W/V was less than about 7.5. In Fig. 6(b) the maximum relative story drifts, calculated ignoring the influence of P-Delta effects, are plotted. The results presented show that inelastic response occurred in stories 1, 2 and 3, with the largest amount of inelastic response occurring in story 1. For the buildings considered, the maximum relative story drifts for story 1 were larger than 2 times the yield relative story drift only when the ratio W/V was greater than 7.5.

Discussion

The results of the studies presented in this paper on the response of shear buildings to the El Centro pulse and the Taft earthquake clearly indicate the influence that P-Delta effects can have on the displacement response of buildings. Provided that the ratios of maximum relative story drift to yield relative story drift were less than about 2 and the ratio of building weight to base shear capacity, W/V, was less than 5 to 10, the influence of P-Delta effects was relatively small. However, when the ratios of maximum relative story drift to yield relative story drift were greater than about 2 and the ratio W/V was greater than 5 to 10, P-Delta effects often significantly increased the displacement response.

In short, for buildings responding elastically or in a slightly inelastic manner to seismic ground motion, P-Delta effects have only a small influence on response. For systems responding in a highly inelastic manner, P-Delta effects often significantly influence the response. On the basis of the results presented in Figs. 3, 4, 5 and 6, it is tentatively suggested that the influence of P-Delta effects on response can be ignored provided the ratios of maximum relative story drift to yield relative story drift are less than 2, the value of the ratio W/V is less than 10 and the maximum relative story drifts are limited to acceptable design values.

STABILITY FACTOR APPROACH

In this section, the stability factor approach is reviewed in the light of the results of the time-history calculations described above. The stability factor approach is an approximate procedure that has recently been recommended for evaluating the influence of P-Delta effects on buildings. The procedure seems to be satisfactory for use in the design of buildings subjected to static loadings when the structure remains elastic under the specified loads.

In applying the stability factor approach, the relative story displacement calculated including the influence of P-Delta effects, Δ, is obtained from the expression (5)

$$\Delta = \frac{V}{k - \frac{\Sigma w}{h}} \tag{1}$$

in which V is the story shear, "k" is the story stiffness, Σw is the sum of the applied gravity loads down to the story considered and "h" is the story height. Equation (1) can be written in a different form if "k" is replaced by V/Δ_o, where Δ_o is the relative story displacement calculated ignoring the influence of P-Delta effects. The expression

$$\Delta = \frac{1}{1 - \frac{\Delta_o}{h}\frac{\Sigma w}{V}} \Delta_o \tag{2}$$

results, where the quantity $1/(1 - \Delta_o \Sigma w/hV)$ is referred to as the stability factor. Of course, elastic behavior is assumed in the derivation of Eqns. (1) and (2).

In Fig. 7 the influence of P-Delta effects on displacement response, calculated using the stability factor approach, is demonstrated. These plots were constructed for the bottom story of a building, for which Σw is equal to the total building weight, W, and are given for three values of the maximum relative story drift, $\psi = \Delta_o/h$.

The response calculated using the stability factor approach, Fig. 7, can be compared to the response calculated using time-history analysis, Figs. 3(a), 4(a), 5(a) and 6(a). (Note, the vertical axis of Fig. 7 is plotted to a larger scale.) It can be seen that the stability factor approach provides reasonable estimates to time-history results for systems responding elastically or in a slightly inelastic manner (low values of the ratio W/V and relatively low values of maximum relative story drift, ψ). On the other hand, when inelastic behavior becomes significant (high values of the ratio W/V and high values of maximum relative story drift), the time-history curves are quite different from the stability factor curves.

Therefore, the stability factor approach, used in the manner described in this study, can only be recommended for buildings responding in an elastic or in a slightly inelastic manner to earthquake ground motion.

PRACTICAL CONSIDERATIONS

In the design of a building to resist earthquake ground motion, the engineer proportions the members of the structure to resist lateral forces specified in building codes. Also, the engineer ensures that story deformations under the specified loading do not exceed certain limits. In this section, the strength and drift properties of well-designed buildings are compared to the properties of the buildings considered in the time-history studies described previously. By making these comparisons, the influence of P-Delta effects on the response of buildings of practical proportions can be assessed.

The minimum design base shears specified by the NBC 1977 (6) vary as a function of the building construction and configuration. A design base shear on the order of 5 percent of the weight of the building is not an unreasonable value. (Recall that the ratio W/V referred to previously is equal to the building weight divided by the base shear capacity.)

In general, the actual strength of a structure will be somewhat larger than the minimum strength needed to satisfy building code requirements. Real buildings are designed for several load combinations, and some members in the structure will likely be oversized for any given load combination. In addition, the usual design practice of using common member sizes as much as possible in a design results in some degree of oversizing. For example, Lu et al. (7) found that moment frames under working gravity loads were capable of resisting a maximum lateral wind equal to two to three times the working wind. For the tall buildings considered in the earthquake response studies of Goel and Berg (8), the base shear capacity of the buildings for inelastic response varied from about 15 to 50 percent of the building weight. The low-rise buildings considered by Montgomery and Hall (9), had base shear capacities between 15 and 65 percent of the building weight. The ten-story building considered by Pekau et al. (10), designed according to an earlier edition of the NBC (6) for Zone 3 seismic exposure, had a base shear capacity of around 15 percent of the building weight.

Consequently, the ratios W/V for buildings of practical proportions will often be smaller than 5 or 10. By comparison with the results of time-history studies presented in the section entitled "Influence of P-Delta Effects on Response", it appears that often well designed buildings will respond to earthquakes of the strengths considered in this study in only a mildly inelastic manner. Therefore, P-Delta effects will have a relatively small influence on the response.

Drift is another consideration. The relative story drift limit recommended in the NBC Commentary (6) is 0.005 under the specified earthquake loads. In the Code (6) it is stated that the deformations under the specified loads should be multiplied by a factor of 3 to give estimates to the deformations during a major earthquake. Thus, it is likely that the drifts of well-designed buildings will be limited to 3(0.005) = 0.015 or less during a major earthquake. If drifts are limited, P-Delta effects are also limited.

In short, the results presented in this paper suggest that, in many cases, P-Delta effects will not significantly influence the seismic response of well-designed buildings of reasonably high strengths provided deformations are limited.

CONCLUSIONS

An evaluation of the influence of P-Delta effects on the earthquake response of buildings has been presented. Although the studies were limited to shear buildings subjected to two earthquake base motions, it is believed that the results can be applied with reasonable confidence to other building types subjected to different earthquake motions. Some of the significant aspects of the results are summarized below.

1. For buildings responding elastically or in a slightly inelastic manner during seismic motion, P-Delta effects have only a small influence on response. In these studies, the influence of P-Delta effects was relatively small for buildings provided that the ratio W/V was less than about 5 to 10 and the ratios of maximum relative story drift to yield relative story drift were less than about 2. For buildings responding in a highly inelastic manner, P-Delta effects can have a significant influence on response.

2. The stability factor approach for estimating the influence of P-Delta effects gave reasonable results when the response was elastic or slightly inelastic. However, the method should not be used for systems responding in a strongly inelastic manner.

3. In many instances, well-designed buildings will be proportioned and constructed in such a way that P-Delta effects do not significantly influence the seismic response.

ACKNOWLEDGEMENT

This study was supported by grants from the General Research Fund, University of Alberta, Edmonton and the Natural Sciences and Engineering Research Council Canada.

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NOTATION

The following symbols are used in this paper:

h = story height;

k = elastic story stiffness;

V = capacity of a story to resist shear; base shear capacity;

W = total gravity load of building (building weight);

Σw = sum of the applied gravity loads down to the story considered;

β = parameter used in the Newmark Beta Method;

Δ = maximum relative story displacement including the influence of P-Delta effects;

Δ_o = maximum relative story displacement ignoring the influence of P-Delta effects;

Δ_y = yield relative story displacement;

ψ = Δ_o/h = maximum relative story drift ignoring the influence of P-Delta effects;

ψ_y = Δ_y/h = yield relative story drift.

Table 1 - Ground Motion Data

Ground Motion	Maximum Ground Motions			Duration
	Displacement (mm)	Velocity (mm/s)	Acceleration (g)	(s)
El Centro pulse	213	331	0.318	6.23
Taft earthquake	67.0	157	0.156	54.4

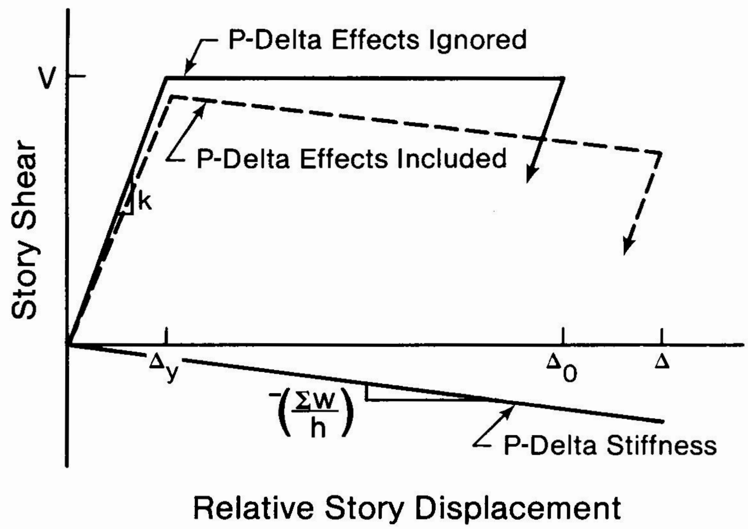


Fig. 1 Story Shear Force-Deformation Relationship

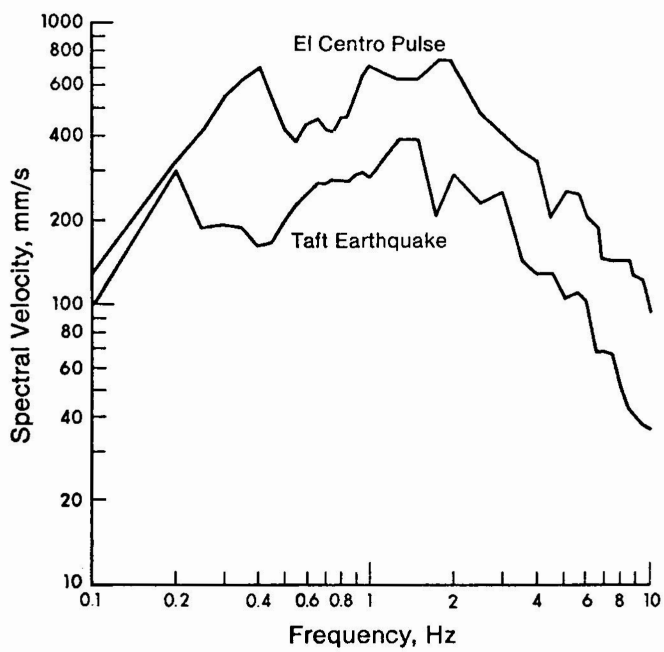


Fig. 2 Elastic Response Spectra, 5 Percent Critical Damping

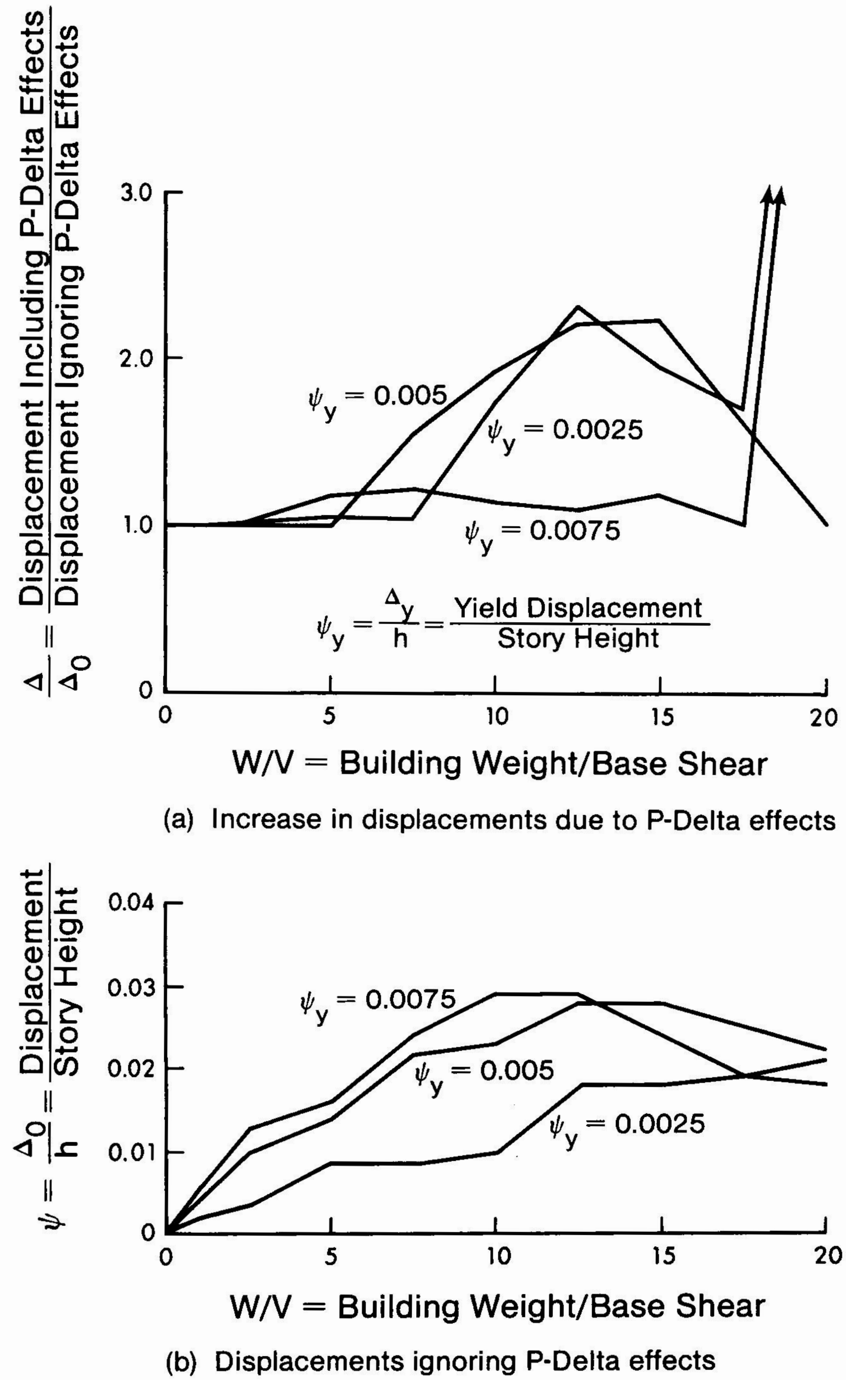


Fig. 3 Influence of P-Delta Effects, One-Story Buildings, El Centro Pulse

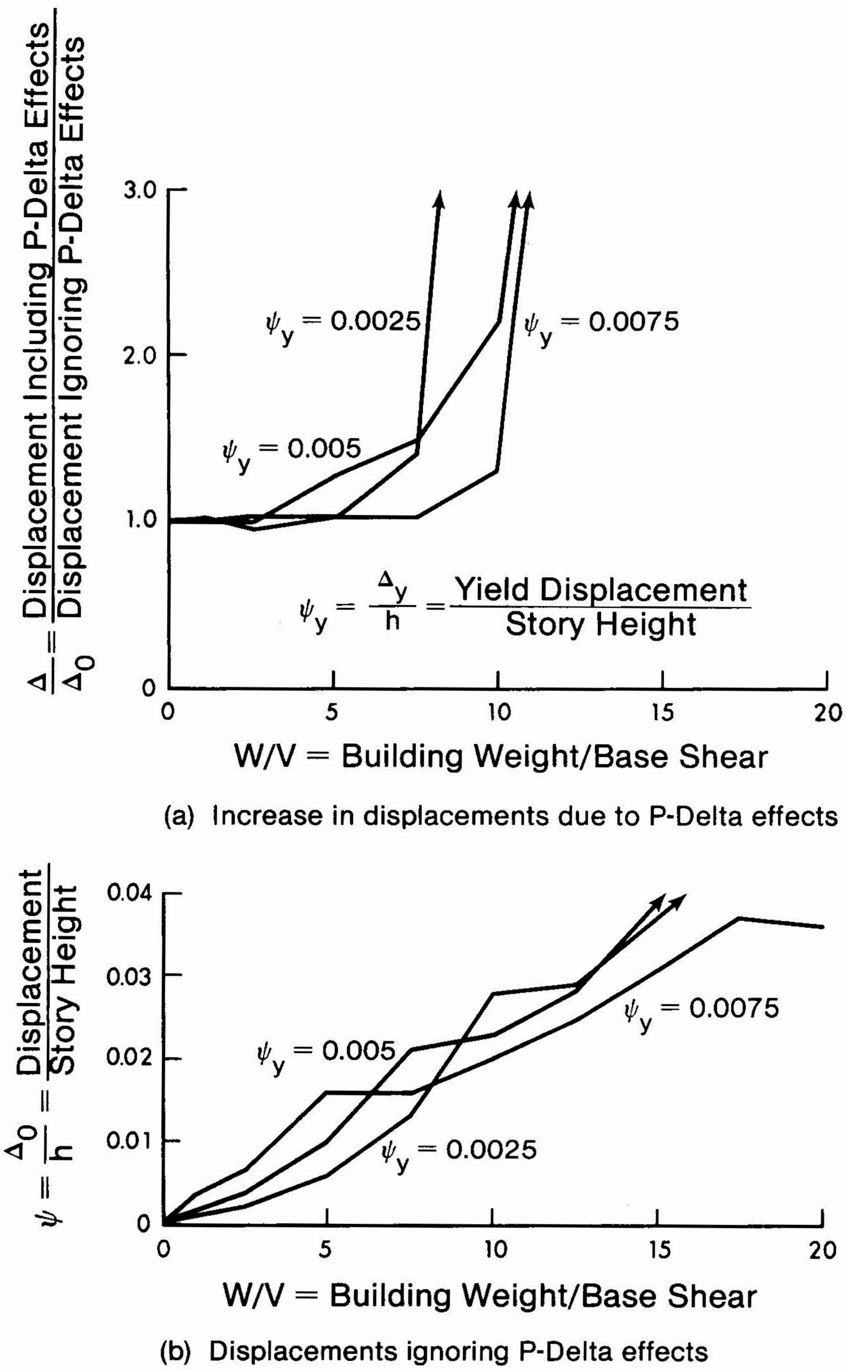


Fig. 4 Influence of P-Delta Effects, One-Story Buildings. Taft Earthquake

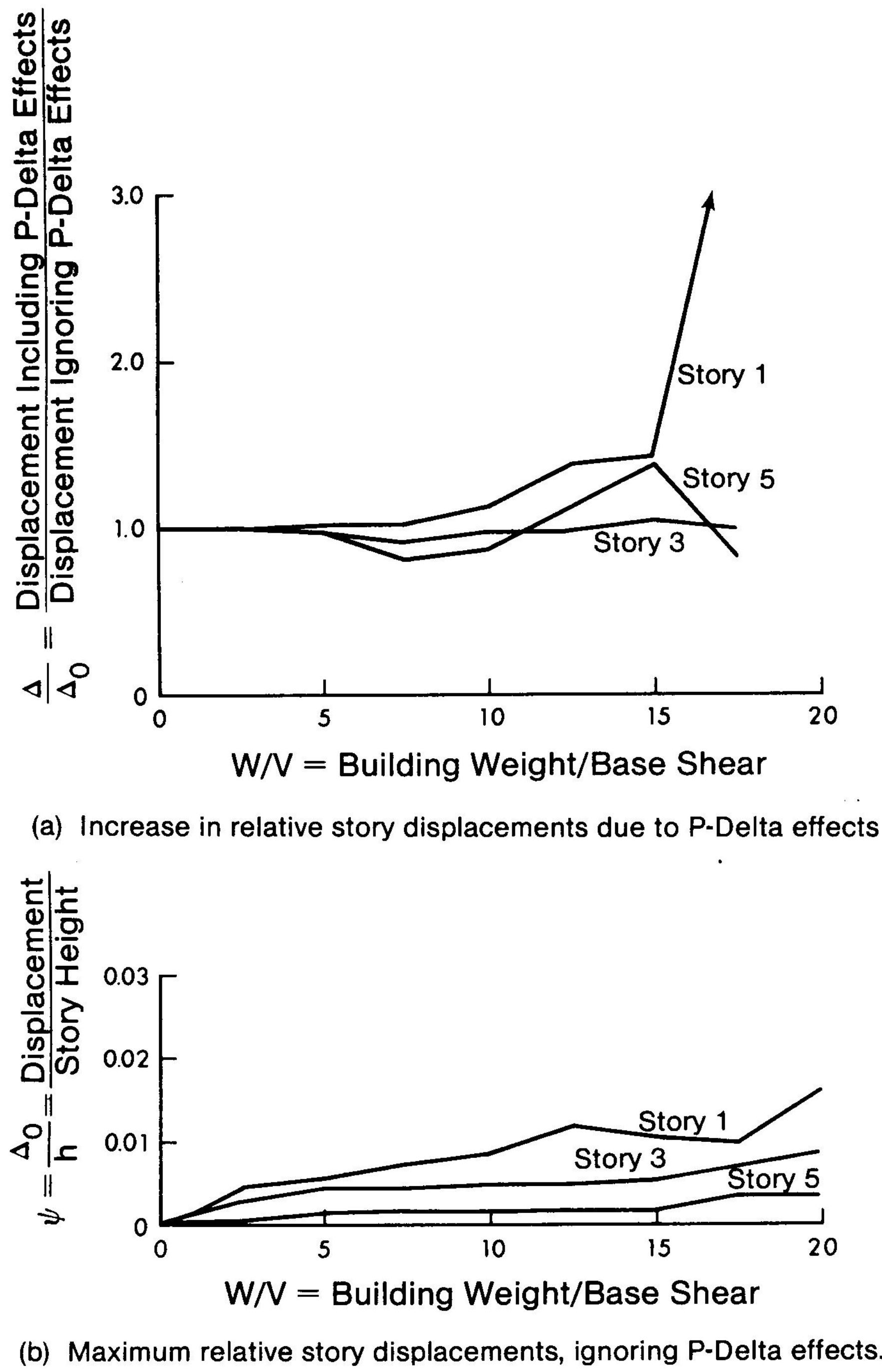
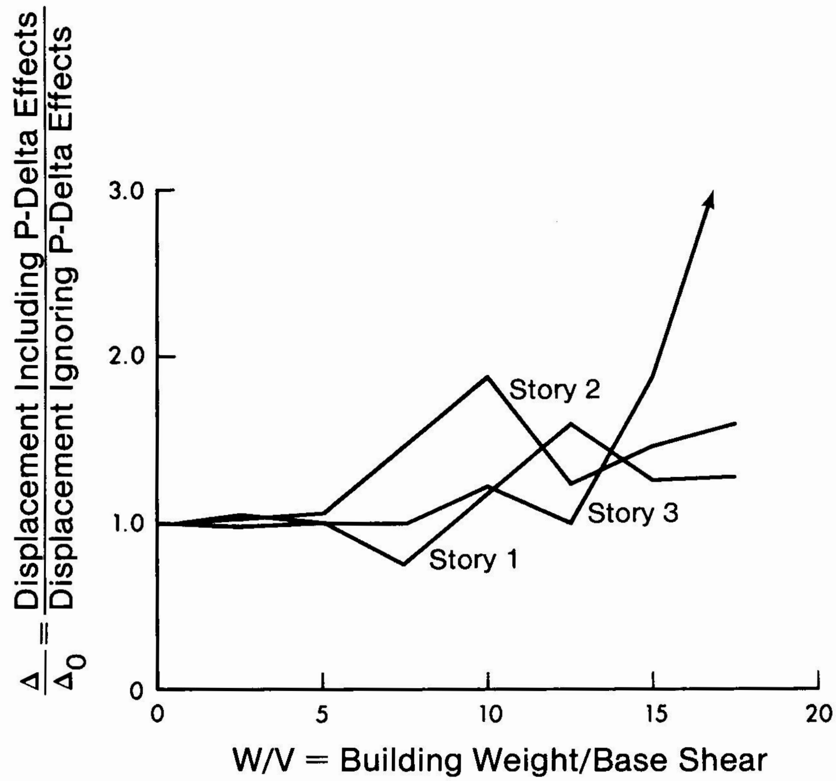
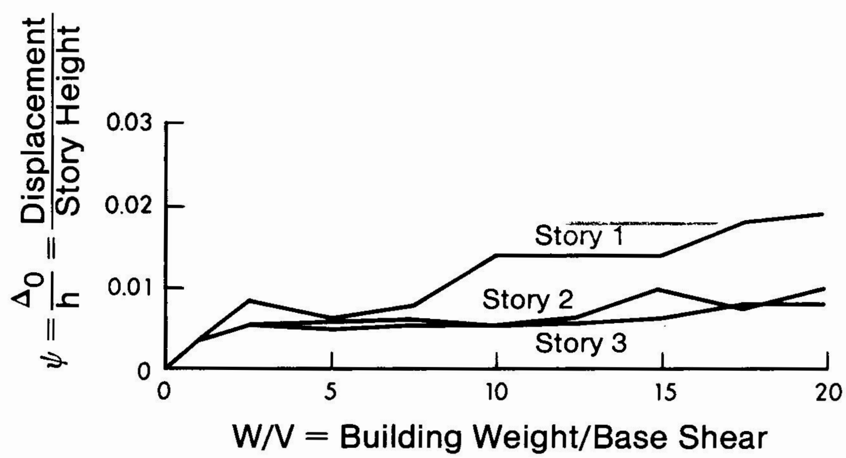


Fig. 5 Influence of P-Delta Effects, Five-Story Buildings, Taft Earthquake

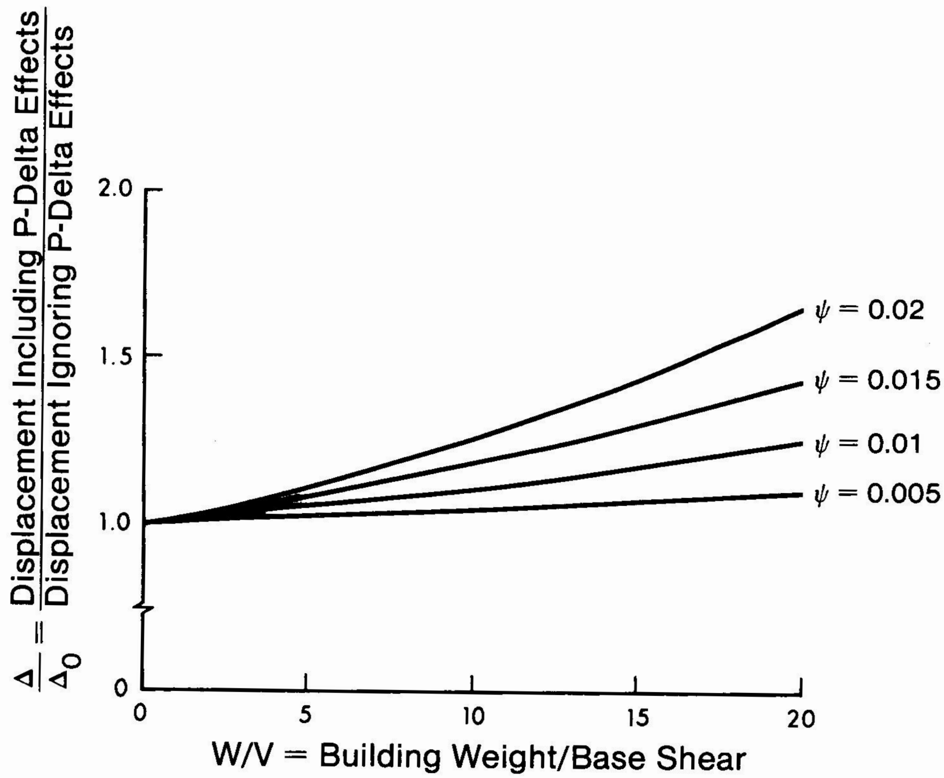


(a) Increase in relative story displacements due to P-Delta effects.



(b) Maximum relative story displacements, ignoring P-Delta effects

Fig. 6 Influence of P-Delta Effects, Ten-Story Buildings, El Centro Pulse



Where $\psi = \frac{\Delta_0}{h} = \frac{\text{Displacement}}{\text{Story Height}}$

$$\Delta = \frac{1}{1 - \frac{W}{V}\frac{\Delta_0}{h}}\Delta_0$$

Fig. 7 Influence of P-Delta Effects, Stability Factor Approach